Use of Octave Filters in the Measurement of Random Noise Spectral Density

Dear Sir:

In a recent journal article DeFelice and Sokol (1976) have discussed some aspects of the experimental problems encountered in measuring noise potentials observed across biological membranes. To determine the noise density at a series of particular frequencies with sufficient convenience, accuracy, and speed to be compatible with the time available to study a given preparation requires some compromises to be made in selecting the spectral resolution used at each frequency interval measured. The conclusion of DeFelice and Sokol is that what they call the integral spectrum, the actual noise spectrum sampled through a simple broad filter, can be measured with more accuracy than the noise spectrum itself, and it can be as useful as the noise spectrum itself.

I have also considered this problem (Strandberg and Hammer, 1975) in preparation for the observation of the current fluctuation noise in the active sodium ion transport current in the toad urinary bladder membrane. A particular experimental arrangement, the result of these considerations, together with the observed current fluctuation noise spectra, have been described in an article published in another journal. The conclusion is that the spectral noise density can be determined with speed, and, contrary to the conclusion of DeFelice and Sokol, with an accuracy comparable to that claimed by DeFelice and Sokol for the integral spectrum. The reason for our contradictory conclusion is simple; DeFelice and Sokol make the common, but unwarranted, assumption that conventional analysis implies that the bandwidth of the filter used to determine the spectral density function must be smaller than the frequencies being observed.

We proceed now to give the details of the considerations which led to our conclusions. A time-stationary, zero-mean, random variable, $V(t)$, will have a mean square value to which the frequencies in the interval f to $f + \mathrm{d}f$ contribute $S_v(f)\,\mathrm{d}f$, so,

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T V^2(t)\,\mathrm{d}t = \int_0^\infty S_v(f)\,\mathrm{d}f. \tag{1}$$

If this random signal is passed through a filter with a power transmission, $T(f)$, the mean square value transmitted will be $\int_0^\infty T(f)S_v(f)\,\mathrm{d}f$. A filter without ripples in the pass band, a Butterworth filter readily available in commercial electrical measuring equipment, will have a transmission for a low-pass filter of the form $T(f) = f_2^{2n}/(f_2^{2n} + f^{2n})$. The index, n, gives the order of the filter, the n-pole character of the filter, or the number of parallel reactances used in a ladder realization of the filter. In a high-pass filter $T(f)$ has the form $f^{2n}/(f_1^{2n} + f^{2n})$ (ITT Engineers, 1968). Cascading a high-pass and a low-pass filter produces a bandpass filter with transmission $T(f) = f_2^{2n}f^{2n}/(f_1^{2n} + f^{2n})(f_2^{2n} + f^{2n})$. The transmission is a maximum, $T(f)$, for $f = (f_1 f_2)^{1/2}$, and $T(f)_{\max} = [(f_1/f_2)^n + 1]^{-2}$. DeFelice and Sokol chose $n = 1$, $f_1 = f_2$, i.e., a single pole filter with a maximum transmission of 1/4. The integrated transmission of the filter defines the power bandwidth, B_n, as,

$$T_{\max} B_n = \int_0^\infty T(f)\,\mathrm{d}f. \tag{2}$$

Hence,

$$B_n = \frac{\pi}{2n\sin(\pi/2n)}\frac{[1 + (f_1/f_2)^n]}{[1 - (f_1/f_2)^n]}(f_2 - f_1). \tag{3}$$

The filter of DeFelice and Sokol thus has a power bandwidth, in their notation, of πl where $l = f_1 = f_2$.

For any filter with f_1 less than f_2, the power bandwidth rapidly approaches $f_2 - f_1$, as n increases. For an octave filter, that is, one with $f_2 = 2f_1$, $B_n/(f_2 - f_1)$ equals 4.70, 1.85, 1.32, and 1.15 as n varies from 1 to 4. The actual cut-off frequencies, f_1 and f_2, thus more accurately define the power bandwidth, B_n, as n increases, and at $n = 4$, B_n is within 15% of $f_2 - f_1$.

To describe a real physical system, $S(f)$ must be limited in form; that is, it must approach zero rapidly enough as f approaches infinity so that the mean square fluctuation of $V(t)$ is finite. It must also approach a constant fast enough as f approaches zero for the same reason. In short, it must be a rational function, without singularities on the positive real frequency axis, varying as f^{-2} as $f \to \infty$, and possessing at $f = 0$ at most an integrable singularity, that is, $S(f) \propto f^{-\epsilon}$, $\epsilon < 1$, near $f = 0$ (Halford, 1968). For finite frequency spans such a spectrum can be approximated to any degree of accuracy by Weierstrass' theorem, by a sum of terms which form a polynomial in f. Hence,

$$S_v(f) = \sum_m S_v^{(m)}(f), \qquad S_v^{(m)} \propto f^{-m}. \tag{4}$$

The transmission integral, $\int_0^\infty T(f)S_v(f)\,\mathrm{d}f$, can be computed exactly, but to make my point, a simpler and more general result that accurately approximates the exact result can be derived by noting that the bandwidth of a filter with $n = 4$ or greater is approximated to 15% or better by $f_2 - f_1$. Hence, for a filter with $n = 4$, such as the one used in the work reported by Strandberg and Hammer (1975), which was produced by a Krone-Hite 330M variable filter (Krohn-Hite Corp., Avon, Mass.), or one of higher order, the transmission integral can be well approximated by

$$\int_0^\infty S_v(f)\,T(f)\,\mathrm{d}f \simeq \int_{f_1}^{f_2} S_v(f)\,\mathrm{d}f, \qquad n \geq 4. \tag{5}$$

In terms of the transmission at the frequency of maximum transmission, $\bar{f}^2 = f_1 f_2$, and for an octave filter, $f_2 = 2f_1$, these terms reduce to

$$\int_0^\infty S_v(f)\,T(f)\,\mathrm{d}f \simeq \Bigg[S_v^{(0)}(\bar{f}) + (2)^{1/2}\ln 2\, S_v^{(1)}(\bar{f}) + S_v^{(2)}(\bar{f}) + \sum_{m=3} (m-1)^{-1}\left(2^{m/2} - \frac{2}{2^{m/2}}\right) S_v^{(m)}(\bar{f})\Bigg](f_2 - f_1), \quad \bar{f}^2 = f_1 f_2. \tag{7}$$

Since $2^{1/2}\ln 2 = 0.980$, and since the $m = 3$ and $m = 4$ coefficients of $S_v^{(m)}$ are 1.06 and 1.17, and since the components $S_v^{(m)}(f)$ for m greater than 4 are usually of vanishing interest, one finds that the output of an octave filter with $n = 4$ or greater is to a good approximation given by,

$$\int_0^\infty S_v(f)\,T(f)\,\mathrm{d}f \simeq B_n S_v[(f_1 f_2)^{1/2}]. \tag{8}$$

The measured mean square transmission of an octave filter thus determines the spectral noise density at the frequency $\bar{f} = (f_1 f_2)^{1/2}$.

The first conclusion therefore is that a broad-band, i.e., octave filter, can be used to determine accurately the spectral density function of a physical noise spectrum.

As for absolute accuracy, it is well known that the statistical average of the square of the output of a filter of bandwidth B_n, measured for a time interval, T, will have a mean square fractional error given by $(B_n T)^{-1}$ to within factors the order of unity (Rice, 1954). For fixed observation time, say some fraction of the time over which necrotizing processes in a biological preparation noticeably change the preparation, for greatest accuracy one must use the greatest filter bandwidth. The bandwidth must also be compatible with its usefulness in allowing one to identify the spectral density from the observations. We have shown that an octave filter allows one to identify the spectral density. Hence any narrower filter will yield less accuracy. This conclusion flatly contradicts the intuitive idea that one often encounters, and which DeFelice and Sokol assume, namely, that to achieve accuracy one must use filters with bandwidths much less than their center frequency.

DeFelice and Sokol determine the mean square deviation of their integral filter and compare it, in their Eq. 40, with the mean square error, $(B_n T)^{-1}$, of a general band-pass filter. They give the ratio as $2\pi l/B$ for measuring frequency independent noise, and as $2\pi l/5B$ for measuring noise varying as f^{-2}. For an octave filter at frequency $(f_1 f_2)^{1/2} = l$, $B = l/(2)^{1/2}$. Hence their result states that the filter error is $[2\pi(2)^{1/2}]^{1/2}$, and $[2(2)^{1/2}\pi/5]^{1/2}$ times that of their integral spectrum. These are factors the order of unity, and are hardly a firm basis for choosing either method for observing noise in biological membranes.

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REFERENCES

DEFELICE, L. J., and B. A. SOKOL. 1976. A new analysis for membrane noise. *Biophys. J.* **16**:827.

HALFORD, D. 1968. A general mathematical model for $|f|^{\alpha}$ spectral density with special reference to flicker noise $1/|f|$. *Proc. Inst. Electr. Electron. Eng.* **56**:251.

ITT ENGINEERS. 1968. Reference data for radio engineers. 5th edition. Howard W. Sams & Co., Inc., Publishers, Indianapolis. 8-1 to 8-4.

RICE, S. O. 1954. Mathematical analysis of random noise. *In* Noise and Stochastic Processes, N. Wax, editor. Dover Publications, Inc., New York. Sec. 3.9 and Appendix 4*c*.

STRANDBERG, M. W. P., and E. I. HAMMER. 1975. Current-fluctuation noise in toad urinary bladder during active transport of sodium ions. *J. Appl. Phys.* **46**:3661.

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Comments on the Analysis of Membrane Noise by the "Integral Spectrum" Procedure

Dear Sir:

Membrane noise reflects events on a microscopic scale and its characterization has the potential of discriminating between various models of ionic conduction. Its study includes three distinct phases: (*a*) acquisition of reliable data, (*b*) analysis of the resulting random process, and (*c*) testing of hypotheses and model fitting. The limited time, T, over which the preparation may yield stationary data and the mixture of several components originating from separate processes are some of the difficulties encountered.

In the context of (*b*), DeFelice and Sokol (1976) have recently discussed a procedure called